Discretization Error Control In the Precise Solids Method (PSM) Part 2

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Editors's Introduction

In the April 2001 issue of Benchmark the authors gave us an insight into workings of Procision when simulating h or p solutions. This article discusses the use of the product in native mode and focuses on quantifying absolute errors.

Procision in native mode

Procision is designed to run using the least possible number of subparts, specifying the desired accuracy in terms of absolute BC errors, enabling stress concentration functions and allowing the iterative solution process to continue until accuracy requirements are satisfied.

Let us now explain why BC errors are called absolute errors as opposed to relative convergence errors? This is because, from model definition, we know the target value for the BC's and from the solution we know the calculated values of the BC's. For example, displacements of the face should be zero if built-in support is defined there; tractions on free surfaces should be zero etc. All the displacement and traction BC's requirements (fig. 11) are met only approximately, but we control the accuracy level. For convenient interpretation of results, BC errors are normalized and expressed as percentage of the maximum corresponding value in the model. Note that it does not take several iterations to calculate BC errors; they can be calculated even if the solution is obtained in just one step. Procision uses iterations to test increasingly complex subpart formulations until the user's specified absolute accuracy is met. In the high stress gradient areas, if it is not economical to control traction BC errors by increasing the p level of subparts and boundary DOF functions, Procision deploys non-algebraic stress concentration functions.

	Requested	Obtained
Strain energy change (relative)	1%	0.28%
Max. displacement BC error (absolute)	3%	1.75%
Max. displacement discontinuity error (absolute)	3%	0.00%
Total RMS traction BC error (absolute)	3%	2.32%
Max. traction BC error (absolute)	30%	12.10%

Note that displacement discontinuity error is 0.00% because our model does not use any splits; displacements within each subpart are continuous.

Figure 12. Solution showing requested and obtained solution accuracy

Iterative solution Process

There are two stages in the iterative solution process. Firstly, the analysis is repeated, each time changing the p order of the volume functions until the change in strain energy is converged to 2%. Having obtained 2% convergence in strain energy, the second stage of solution begins; BC errors take over the control of further iterations. The boundary DOF functions assume the p order required to bring displacement BC errors and displacement discontinuity errors within the user's requested accuracy and stress concentration functions are deployed to reduce explicitly BC traction errors. At the same time, Procision continues upgrading volume functions watching for strain energy convergence, which must be below 1% in the final solution.

Technically, the iterative solution could be controlled either only by the relative strain energy error or only by BC errors. However numerical experiments demonstrated that BC error control is best activated only after the solution has became "good enough" in the sense of strain energy convergence, what may be identified as an approximate solution in displacements. The solution process is split in two phases to minimize the run time.

Figure 12.shows a table of the requested accuracy against solution accuracy in true Procision mode. The definitions and explanation of these terms can be found in Figure 16.

DISPLACEMENT BC REQUIREMENTS

On surfaces where boundary conditions are explicitly specified, those boundary conditions should be satisfied exactly

Splits should not introduce discontinuity of displacements

TRACTION BC REQUIREMENTS

Equal to the applied load on loaded surfaces

Equal to reactions on surfaces where supports are defined

Equal and opposite on two surfaces created by split

Zero on all non-loaded surfaces

Figure 11. Boundary Condition Requirements & Errors

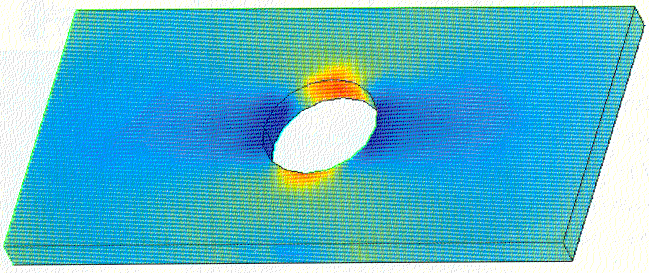

Figure 13: Procision Solution with Stress Concentrations
Max Principal sress = 393 MPa

The maximum principal stress results are presented in fig. 13 Note that the simple geometry indeed permits the entire model to be represented as "one big subpart" without any splits.

Conditions for using BC errors as tools of solution quality control

Readers may now ask why traction BC errors are not used to control the solution quality in FEA? The authors think their are two major reasons.

The first is that even though traction BC error can be evaluated and displayed in FEA, this error does not fully describe discretization error in FEA. Traction BC error provides reliable information on the quality of solution only if equilibrium requirements are satisfied, as is the case with subparts in PSM but not with elements in FEA. In subparts in equilibrium, traction errors represent the discretization error as a whole. Those traction errors can actually be conceptualized as additional or "parasite" loads.

The second reason is that traction BC errors must be calculated from stresses on the surface of a part, while in FEA stresses themselves are calculated unreliably on the surface. Usually stresses are calculated in special points in the volume of a finite element (below the surface), then extrapolated to the surface and, finally, averaged among the elements. So it would be meaningless to calculate tractions error in FEA if those traction errors themselves are calculated using unreliable surface stress results.

Conclusions

We used a step-wise approach to introduce the concepts of discretization error control in Procision. First, we demonstrated how to make Procision behave as an h code. In several steps, we successfully replicated the h convergence process. Encouraged by the success of h convergence example, we then proceeded to manipulate Procision settings so it performs as a p code. This time the results showed the lack of convergence because we were able to control the p level of volume functions, but not of boundary DOF functions. The Procision interface simple does not offer the option of controlling the boundary DOF functions. Even though this option could easily be added, it would be completely unnecessary.

While forcing Procision into h and p mode, we introduced major concepts of PSM and of discretization error analysis in PSM, showing that it is based on absolute BC error and made

Figure 14: More Complex Geometry which can be analyzed in Procision without defeaturing and/or idealisations, showing the division into subparts.

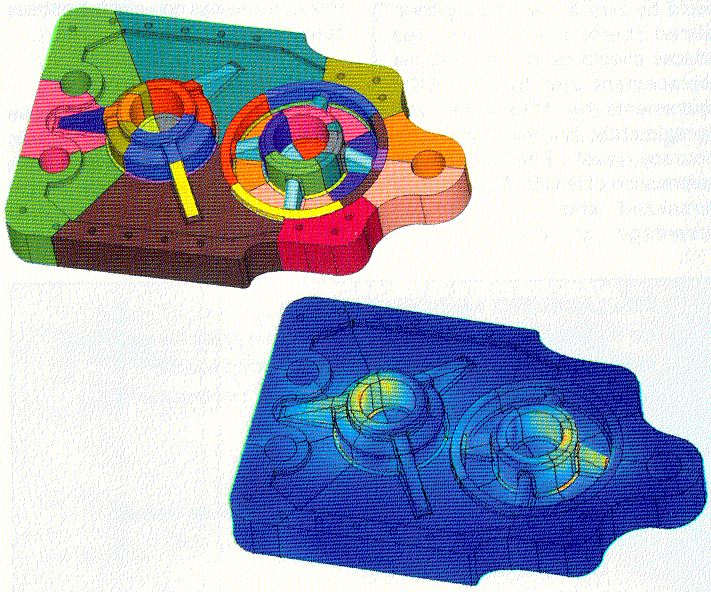

Figure 15: Von Mises stress results

possible by the exact fulfillment of equilibrium conditions using volume functions. Finally we demonstrated the native Procision mode, and run the tensile strip the way it should have been run to begin with. We would never advise Procision users to run it in h or p modes, it was never intended to replicate the behavior and functions of FEA software.

Our tensile strip model with hole did not do justice to Procision capabilities, which are best used on complex solid geometries.

As a meshless technique, Procision is not subjected to any of the restrictive meshing requirements typical for FEA models. It solves geometries of any complexity without defeaturing, idealization and clean up (fig. 14, 15). These abilities eliminate the need for interfacing between CAD and analysis geometry and greatly affect the way analysis can be implemented in the design process.

We hope to return to this aspect of the implementation of analysis in a future article.

Figure 16: Definition of Terms

Maximum displacement BC error
Absolute error measure. The default 3% value means that the displacement BC or the displacement discontinuity error anywhere in the model must be less than 3% of maximum displacement. Enabled by default.

Total RMS traction BC error change
Relative error measure describing the change in averaged (RMS) traction BC error in the entire model, as calculated between two consecutive iterations. This error measure dates back to early Procision releases the when absolute BC errors were not available. It is still provided only for conceptual compatibility with earlier releases.

Local RMS traction BC error change
This is also a relative error measure describing the highest change of averaged (RMS) traction BC error in individual subparts, calculated between two consecutive iterations. It is disabled for the same reasons as the above total RMS traction BC error change.

Total RMS traction BC error
Traction BC error averaged across all subparts but not on split surfaces. This absolute error measure is enabled by default and set at 3%.

Maximum traction BC error
Maximum traction BC error anywhere, in any subpart, including split surfaces. This is also an absolute measure, enabled and set by default at 30%. Even though the default value 30% may look high, we need to remember that 30% error occurs in one, usually very small, portion of the model and usually far away from areas of interest as assured by stress concentration functions.

Accuracy level
This is the minimum and the maximum polynomial order of volume functions. By default $p_{min} = 3$, $p_{max} = 9$. pmax may be increased to 12 at the expense of solution time.

Maximum number of passes
Maximum number of iterative steps set by default at 15. This maximum number of steps is almost never reached. Usually the solution completes in 5 to 7 passes. This condition is provided to prevent an "infinite loop" in some rare cases of ill-defined models.

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Editor's Note: Indeed Paul Kurowski presented a paper, titled "The Precise Solids Method Implemented in the Design Process", at the recent NAFEMS World Congress, in Lake Como, Italy. Copies of the conference proceedings can be purchased directly from the NAFEMS office.

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